# Titles and abstracts

## 1 Introductory lectures

### Marco Golla - A quick introduction to Khovanov homology

Khovanov homology is a categorification of the Jones polynomial. I will give a quick introduction to them, explaining in the process what it means to categorify a polynomial.

### Rémi Leclercq - Introduction to Morse and Floer homologies

I will start by reviewing the construction of Morse homology, outlining its main steps without going into technical details. I will then present Floer homology, following the same general framework.

### Maxime Moscatelli - Classifying space of the flow category associated to a Morse function

Given a topological category $C$, we can construct its classifying space as a CW-complexe whose $k$-cells represents commutative diagrams with $k$ vertices and whose attaching maps are given by the composition in $C$. On the other hand, given a Morse function $f$ on a closed Riemannian manifold $M$, we can define its flow category $C_f$ as a category whose objects are the critical points and the set of morphisms are piecewise gradient flow-lines. Cohen, Jones and Segal showed that the classifying space $C_f$ has the homotopy type of $M$ and is even homeomorphic to $M$ when $f$ is Morse-Smale. In this talk, I will present the construction of classifying space of a general flow category and the proof of Cohen–Jones–Segal, and give some constructions examples of classifying spaces.

## 2 Mini-courses

### Marithania Silvero - An overview on Khovanov spectra

In 2000 Khovanov introduced a homological link invariant which categories Jones polynomial. A few years later, Lipshitz and Sarkar introduced a stable homotopy refinement of Khovanov homology: given a link diagram, they provided a method to construct a suspension spectrum whose cohomology is the Khovanov homology of the link represented by D.

In this course we will review the main aspects of Khovanov spectra, including their construction(s) and some of their nice properties. If time permits, we will also present how these constructions can be simplified under suitable conditions.

### Noah Porcelli - Flow categories in symplectic Floer homotopy theory

I'll discuss a geometric approach to Floer homotopy theory due to Abouzaid and Blumberg, based on bordism theory, and how this relates to holomorphic curve theory.

## 3 Short talks

### Giovanni Framba - Slice genus bounds for strongly invertible knots

A strongly invertible knot is a knot embedded in the 3-sphere together with a suitable involution of the ambient space that preserves the knot. We introduce a variant of grid homology that incorporates these symmetry properties. Using this framework, we define a pair of invariants that provide a lower bound for the slice genus of equivariant cobordisms between strongly invertible knots.

### José Narbona-Valiente - Artin–Tits groups acting on symplectic manifolds

Given a graph $\Gamma$, there are two rather different ways of encoding its combinatorics. On the one hand, we can build a symplectic manifold from $\Gamma$ by assigning to each vertex a copy of $T^*S^n$, with the edges telling us when to "plumb" two such cotangent bundles together. The result is a Weinstein manifold, which we call $P^{2n}(\Gamma)$. On the other hand, $\Gamma$ also gives rise to an important group called its Artin–Tits group $A(\Gamma)$. These two constructions are closely related. Each zero-section in $P^{2n}(\Gamma)$ has an associated compactly supported symplectomorphism, called a generalised Dehn twist. These twists correspond to the standard generators of $A(\Gamma)$ and satisfy the relations defining the group. Hence, we have a symplectic action of $A(\Gamma)$ on $P^{2n}(\Gamma)$, or more precisely, a homomorphism from $A(\Gamma)$ to the symplectic mapping class group of $P^{2n}(\Gamma)$. The main aim of the talk is to explain this construction carefully: how the plumbing and the group are built, what a generalised Dehn twist is, and why these twists satisfy the relations in the group presentation of $A(\Gamma)$. Time permitting, I will also discuss the fundamental question motivating this setup: what does this action actually remember? Does the geometry introduce unexpected relations between the Dehn twists, or can higher-dimensional symplectic geometry faithfully remember the combinatorics of the graph?

### Sergio Garcia Rodrigo - Quantum Annular Homology and Bigger Burnside Categories

As part of their construction of the Khovanov spectrum, Lawson, Lipshitz and Sarkar assigned to each cube in the Burnside category of finite sets and finite correspondences, a finite cellular spectrum. In this talk, we extend this assignment to cubes in Burnside categories of infinite sets. This is applied to the work of Akhmechet, Krushkal and Willis on the quantum annular Khovanov spectrum with an action of a finite cyclic group, to obtain a quantum annular Khovanov spectrum with an action of the infinite cyclic group.

### Simone Tagliente - On smooth structures over 4-manifolds with fundamental group of even order

We show that any topological, closed, oriented, non-spin 4-manifold with cyclic fundamental group of order $4k$ and $b_2^+$, $b_2^-$ at least 15 has either none or infinitely many distinct smooth structures. This extends a result of Baykur-Stipsicz-Szabó. We distinguish the diffeomorphism types of these exotic manifolds using the Seiberg-Witten invariants of their universal covers, which we construct by assembling simply-connected building blocks and then performing equivariant torus surgeries. Time permitting, we discuss how to construct exotic irreducible manifolds with fundamental group isomorphic to $\mathbb{Z}_2 \times G$ for any finite group G.

### Maartje Wisse - The Dowlin Spectral Sequence: The Good, The Bad and the Ugly

In 2018, Nathan Dowlin proved that there exists a spectral sequence from Khovanov homology to Knot Floer homology, a conjecture of Rasmussen that had been open since 2005 (the good). In this talk I will explain the details of the construction, as well as the current barriers to its use in the field (the bad) and my own work on making the construction more accessible and usable (in progress, the ugly).

### Lin Zheng - Building the Analytic Foundations of Hamiltonian Floer Homotopy: The Smooth Structure of Compactified Moduli Spaces

In the construction of the Floer homotopy type, a fundamental analytic prerequisite is endowing the compactified moduli space of Floer trajectories with the rigorous structure of a smooth manifold with corners. This talk provides a systematic exposition of this geometric and analytic construction.

We begin by recapping the smooth structure of the uncompactified moduli space and then introduce complete systems of transverse hypersurfaces to extract explicit local coordinates.

To understand the boundary—which consists of broken Floer trajectories—we review the forward gluing construction, mapping a broken trajectory and large time parameters to a unique smooth trajectory near the original broken one.

The core of the talk will focus on the reverse process: constructing "boundary-hugging" local charts (collar neighborhoods) on the moduli space near these broken trajectories. We will demonstrate that any smooth trajectory approaching a broken one in the Gromov-Floer topology must arise from the gluing process. The technical difficulty here is to justify that the closeness in the sense of Gromov-Floer topology implies a uniform $C^0$ closeness.

Finally, we show how to naturally extend these internal collar coordinates to the compactification boundary. By introducing a gluing profile ($r = 1/T$) and leveraging the exponential decay estimates of the gluing map, we prove that all transition maps between these charts extend smoothly to the $r = 0$ boundary strata. This resolves the manifold-with-corners structure on the compactified moduli space, ultimately paving the way for defining stable framings and flow categories.